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RECURSIVE ESTIMATION TECHNIQUES FOR DETECTION OF SMALL OBJECTS IN INFRARED IMAGE DATA*

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ABSTRACT

This paper describes a recursive detection scheme for point targets in infrared(IR) images. Estimation of the background noise is done using a weighted autocorrelation matrix update method and the detection statistic is calculated using a recursive technique. A weighting factor allows the algorithm to have finite memory and deal with nonstationary noise characteristics. The detection statistic is created by using a matched filter for colored noise, using the *estimated* noise autocorrelation matrix. The relationship between the weighting factor, the nonstationarity of the noise and the probability of detection is described. Some results on one- and two-dimensional infrared images are presented.

1. INTRODUCTION

A well known technique of detecting objects in noise is matched filtering. For signal with known shapes, for a fixed integration time, matched filtering has been applied in the presence of white and colored noise[1][2]. In the presence of colored noise most detection schemes use a prewhitening filter [1][2][3][4] to whiten the noise, before applying a matched filter to get a detection statistic. This prewhitening filter may be adaptive or non adaptive depending on the amount of *a priori* knowledge available about the corrupting noise process. For detection in unknown and possibly nonstationary noise, an adaptive filter can be used, leading to a two stage formulation with a prewhitening filter followed by a matched filter[3][4][5]. If the object of interest is known to be small, occupying less than a pixel, this shape is then defined by the point spread function of the optics which is known in advance. In this case, or in the case where the shape of the signal is known precisely, the two steps of whitening the noise and matching to the signal can be combined into a single step.

In Section 2 we describe the algorithm for the estimation of noise and the calculation of the detection statistic as a single step algorithm. In section 3 we describe the

performance of this recursive detection technique in the presence of non stationary noise. Section 4 describes the application of this detection technique to infra red images with two dimensional signals.

2. RECURSIVE ESTIMATION AND DETECTION

For a known signal in colored noise, the detection statistic is given by[1][2]

$$g = s^T \Phi^{-1} x \qquad (1)$$

where g is the detection statistic, Φ is the autocorrelation matrix of the noise, s a $p \times 1$ vector is the known signal to be detected, x a $p \times 1$ vector is the input data and p is the length of the known signal or the time of integration in classical detection. g is compared to a threshold, g_0, and a decision on the presence or absence of the signal, s, is made accordingly.

If the noise is unknown, Φ must be estimated from the received data. If the time of integration, n is fixed, a maximum likelihood estimate of the noise autocorrelation matrix is given by [7]

$$\Phi = \sum_{k=0}^{n} x(k)x(k)^T \qquad (2)$$

Often, detection is done with a finite length, known signal template and a continuous stream of input data. s is a $p \times 1$ vector, but a decision on the presence or absence of s has to be taken at the time $n > p$ and at every time instant after that. This is known as *the recursive detection problem.*

In such a case, the detection statistic at each time n is given by

$$g(n) = s^T \Phi^{-1}(n)x(n) \qquad (3)$$

and the estimate of the noise characteristics can be improved as more and more samples of the incoming data become available. In particular, if the signal is assumed to have very low energy as compared to the noise (as is the case with point signals in spatially extended noise), Eq. 2 can be used to estimate the noise recursively.

For nonstationary noise applications, a weighted estimate can be used, leading to

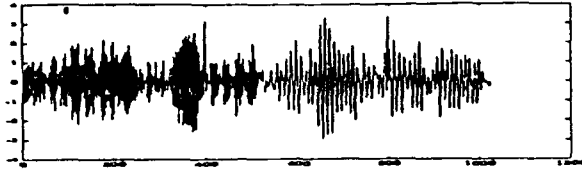$$\Phi(n) = \sum_{k=0}^{n} \lambda^{n-1} x(k)x(k)^T \qquad (4)$$
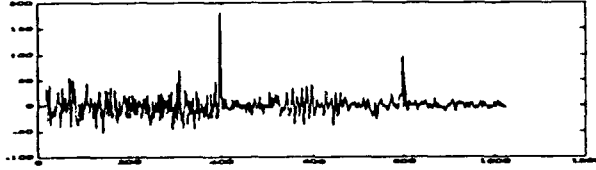
Figure 1: Input: Non-stationary

Figure 2: Detection Statistic: weighted

where λ is a weighting factor. If the noise is nonstationary and $\lambda < 1$, this estimation technique will be able to track the non stationarities in the noise and consequently give a more precise detection performance than a fixed matched filter. $\lambda = 0$ corresponds to an instantaneous estimate of the noise and $\lambda = 1$ corresponds to the maximum likelihood estimate for a stationary noise process. The tracking behaviour of such an estimation process for $0 < \lambda < 1$, for one dimensional adaptive prediction has been studied in [8] and others. In section 3, the detection performance of such an estimation procedure is described.

Eq. 4 can be expressed as a recursion equation, to get a recursive update equation for Φ^{-1} ie,

$$\Phi^{-1}(n) = \lambda^{-1}\Phi^{-1}(n-1)$$
$$- \left(\frac{\lambda^{-2}\Phi^{-1}(n-1)\mathbf{x}(n)\mathbf{x}^T(n)\Phi^{-1}(n-1)}{1 + \lambda^{-1}\mathbf{x}^T(n)\Phi^{-1}(n-1)\mathbf{x}(n)} \right). \quad (5)$$

This is similar to the update equation used for updating the weights of a recursive least squares adaptive filter [7].

This leads to a detection statistic given by

$$g(n) = \frac{\lambda^{-1}\mathbf{s}^T\Phi^{-1}(n-1)\mathbf{x}(n)}{1 + \lambda^{-1}\mathbf{x}^T(n)\Phi^{-1}(n-1)\mathbf{x}(n)} \quad (6)$$

Simulation results for this weighted decision statistic are shown for one dimensional nonstationary data. Fig. 1 shows the non stationary input to such a detection filter. The noise used, was a second order AR noise model, and the coefficients of the model were allowed to change abruptly at the sample point 512. Two signals were embedded in this noise, one at sample point 400 and another at 800. Fig. 2 shows the output from such a detection filter using Eq. 5. Fig. 3 shows the detection statistic that would be obtained if the noise were assumed to be stationary, and the estimation of the autocorrelation matrix was done without any weighting. This corresponds to $\lambda = 1$. It is seen that using the complete history of the noise process to do a *maximum likelihood estimate* of the noise autocorrelation matrix does not allow the matched filter to *adapt* to any changes in the input noise and detection performance
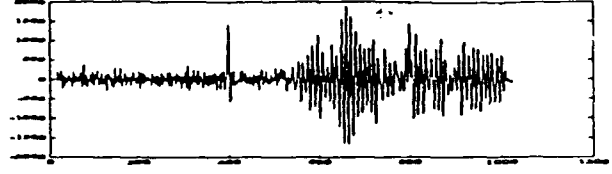

Figure 3: Detection Statistic: without weighting

degrades with any change in the noise statistics. As seen in Fig. 2, using a value of $\lambda = 0.99$ leads to the matched filter being able to adapt to the changing noise statistics and hence provide a detection statistic corresponding to the signal of interest.

3. DETECTION STATISTICS

In this section we study the change in the probability of detection for this recursive estimation technique. The probability density function of the data under H_0 (signal absent) and H_1 (signal present) is assumed to be Gaussian. The variance under both test hypotheses are equal. The mean under H_0 is zero, while the mean under H_1 is known[2] to be $m = s^T\Phi^{-1}s$. The probability of detection, for a stationary process, then becomes

$$P_d = \frac{1}{2}erfc\left[\frac{g_0 - m}{\sigma\sqrt{2}} \right] \quad (7)$$

where, σ is the variance of the process, m is the mean under H_1, g_0 is the threshold used, and $erfc()$ is defined as

$$erfc(x) = \frac{1}{2\pi}\int_x^\infty e^{-x^2}dx \quad (8)$$

Similarly the probability of false alarm is defined as

$$P_{fa} = \frac{1}{2}erfc\left[\frac{g_0}{\sigma\sqrt{2}} \right] \quad (9)$$

For a non-stationary process, assume that at the $n - 1$ sample, the estimation process is exact and the autocorrelation matrix used in the detection process is exact. Then, the following quantities can be found exactly as

$$g^*(n-1) = s^T\Phi^*(n-1)^{-1}\mathbf{x}(n-1) \quad (10)$$

$$m^*(n-1) = s^T\Phi^*(n-1)^{-1}s \quad (11)$$

$$\sigma^*(n-1) = \left(s^T\Phi^*(n-1)^{-1}s\right)^{\frac{1}{2}} \quad (12)$$

$$P_d^*(n-1) = \frac{1}{2}erfc\left[C_f - \frac{m^*(n-1)}{\sqrt{2}\sigma^*(n-1)} \right] \quad (13)$$

$$P_{fa}^*(n-1) = \frac{1}{2}erfc[C_f] \quad (14)$$

where the * implies the "optimum values" obtained by the use of the exact autocorrelation matrix. Here C_f is a constant defined by the required false alarm rate as $C_f = erfc^{-1}(2P_{fa})$.

Assuming the detector is designed to operate at a constant false alarm rate, ie.,$P_{fa}(n-1) = P_{fa}(n)$ the new

2

"optimum" probability of detection in nonstationary noise is given by Eq. 15

$$P_d^*(n) = \frac{1}{2} erfc \left[C_f - \left(\frac{s^T \Phi^*(n)^{-1} s}{2} \right)^{\frac{1}{2}} \right] \qquad (15)$$

where, $\Phi^*(n)$ is the *exact* autocorrelation matrix at the nth sample.

The corresponding probability of detection based on the estimated autocorrelation matrix is

$$P_d(n) = \frac{1}{2} erfc \left[C_f - \left(\frac{s^T \Phi(n)^{-1} s}{2} \right)^{\frac{1}{2}} \right] \qquad (16)$$

where, $\Phi(n) = \lambda \Phi(n-1) + x(n)x^T(n)$.

To model the nonstationarity of the noise in the input data, we assume a change in the inverse of the autocorrelation matrix as

$$\Phi(n)^{-1} = \Phi(n-1)^{-1} + \Delta \Phi \qquad (17)$$

with $\Delta \Phi$ describing the amount of non-stationarity in the noise process.

Further, assuming $\Delta \Phi$ to be small, and expanding the square root in a Taylor series

$$P_d^*(n) \approx \frac{1}{2} erfc[C_d^*] \qquad (18)$$

where

$$C_d^* = C_f - \frac{m^*(n-1)}{\sqrt{2}\sigma^*(n-1)} - \frac{\delta \sigma^*(n-1)}{2m^*(n-1)} \qquad (19)$$

and $\delta = s^T \Delta \Phi s$.

The probability of detection based on the estimated autocorrelation matrix can be similarly found to be

$$P_d(n) \approx \frac{1}{2} erfc[C_d^* + \Delta_d] \qquad (20)$$

where the factor by which this probability of detection differs from the *optimum* is given by

$$\Delta_d = \frac{1}{\sqrt{2}} \frac{m^*(n-1)}{\sigma^*(n-1)} \left(1 - \frac{1}{\sqrt{\lambda}} \right) + \frac{1}{2} \frac{\sigma^*(n-1)}{m^*(n-1)} (\delta + \Delta) \qquad (21)$$

and

$$\Delta = \frac{\lambda^{-2}[s^T \Phi^*(n-1)^{-1} \Phi(n)\Phi^*(n-1)^{-1}s - \lambda m^*(n-1)]}{1 + \lambda^{-1} x^T \Phi^*(n-1)^{-1} x} \qquad (22)$$

Equations 20 21 and 22 define the relationship between the probability of detection (P_d), the rate of non stationarity of the noise $(\Delta \Phi)$ and the weighting factor of the weighted update (λ). Thus a proper choice of λ depends on the autocorrelation matrix of the noise, and can influence the probability of detection.

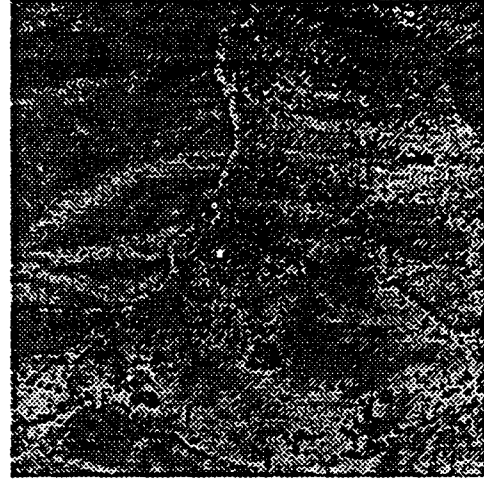

Figure 4: Input: Object Embedded in Infrared Image Data

4. TWO DIMENSIONAL DATA

Infrared image data typically consists of sequences of two dimensional images in one or more spectral bands. Typical applications range from the detection of very small tumors in medical images to the detection of long range airborne targets. The signal of interest in the application considered is a small point source of light distributed around a single pixel according to the point spread function of the sensor optics. This point spread function is typically known. For example Chan et. al. [9] show that for a Gaussian point spread function and a small sub-pixel target, the *known* signal can be modeled as

$$s(x,y) = \Gamma e^{-\frac{(x-x_0)^2 + (y-y_0)^2}{\sigma^2}} \qquad (23)$$

where Γ is the maximum height of the signal, (x_0, y_0) is the spatial position of the signal and σ, the rate of spatial decay of the signal. This signal is typically embedded in spatially extended nonstationary clutter

Since such data maybe highly non-stationary across the image, a two dimensional version of the recursive detection procedure developed in section 2 can be formulated by ordering the data lexicographically as a vector. The autocorrelation matrix for such an ordered data set then becomes a block toeplitz matrix[10]. Such a matched filter was used in a noise canceler structure for multi-spectral images and a typical input image and the corresponding output detection statistic are shown in Fig. 4 and Fig. 5. The noise autocorrelation matrix was estimated from one channel of the multi-spectral data set, while the matching was done on another channel.

The test signal in this case was uncorrelated between the reference and the primary images. This enabled an estimation of the noise statistics and led to considerably whitening of the noise in the output detection statistic as can be seen in Fig. 5.
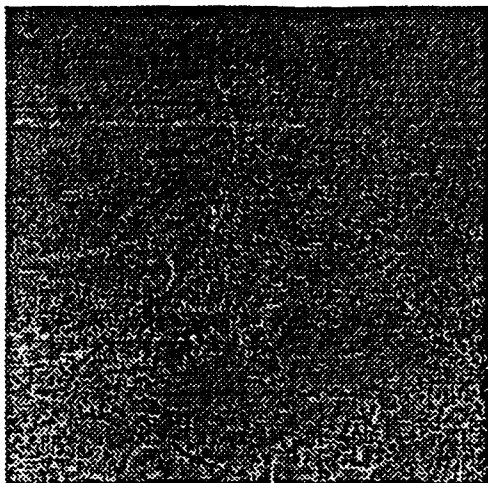
The Signal to Noise Ratio(SNR) for a point signal in

Figure 5: **Detection Statistic: Recursive**

such an image was defined as the

$$SNR = 10 \log_{10} \left(\frac{\gamma_i^2}{\sigma_i^2} \right) \quad (24)$$

where γ_i is the intensity value of the brightest pixel in a given region of interest and σ_i^2 is the per pixel noise energy in the same region defined as

$$\sigma_i^2 = \frac{1}{M^2} \sum_i \sum_j x^2(i,j) \quad (25)$$

M^2 is the size of the region of interest. For the figures shown, a window size of 3x3 (corresponding to $p = 9$ in Eq. 3) was chosen to run this recursive matched filtering algorithm. The region of interest around the target was chosen to be of size 9x9(viz. in Eq. 25 $M = 9$). The input SNR was then found to be 2.67dB and the output SNR was found to be 9.70dB.

The removal of correlated clutter from the input can be clearly seen from the histograms of the input and output images. Fig. 6 shows the fractional distribution of the pixel intensity values in the input image. Note the non-gaussian distribution of the pixel intensities, caused by the nonstationarity of the clutter in the image. Fig. 7 shows the fractional distribution of the pixel intensity values for the output detection statistic. Clearly the different sections of the correlated clutter which caused the input distribution to look non-gaussian have been eliminated by the noise canceler structure and the output detection statistic has a distribution much closer to gaussian.

In summary, the recursive estimation procedure when coupled with the colored noise matched filter leads to a detection procedure that can adapt to nonstationarity in the background clutter.
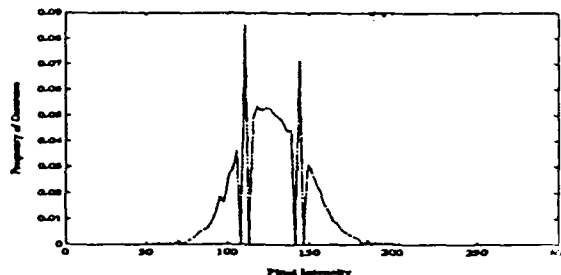
ACKNOWLEDGEMENTS

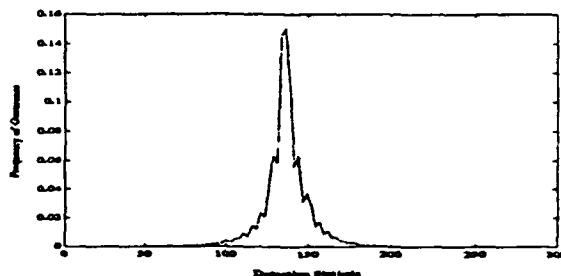Figure 6: **Input Pixel Intensity Distribution**

Figure 7: **Detection Statistic Distribution**

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